MA.7.G.4.1 Determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and apply these relationships to solve problems.

**Perimeter**
Perimeter is the distance around a polygon. Simply find all the sides of the polygon and then add. Be careful, they often only tell you some of the sides, you must find the missing sides. It often helps if you draw the polygon.

Rectangle with length of 4 meters and width of 2 meters.

\[ P = 4 + 4 + 2 + 2 = 12 \text{ m} \]

***Remember, you should be adding the same amount of numbers as sides in the polygon. If you are finding the perimeter of any quadrilateral, you will add 4 sides. For a pentagon, you will add 5 sides. For a hexagon, you will add 6 sides...***

***Remember also that a regular polygon has all sides congruent. So if you are working with a square, all 4 sides will be the same length and you will only be given one side.***

**Area**
Area is the space inside of a shape. The formulas for area will be on the reference sheet. If you know the shape and the formula, just fill in the numbers and solve.

Area of rectangle/parallelogram: \( A = bh \)
Area of triangle: \( A = \frac{1}{2}bh \) or \( A = \frac{b \times h}{2} \) (don’t forget to take half of the base times height)

Area of a trapezoid: \( A = \frac{1}{2}h(b_1 + b_2) \) (bases are the parallel sides of the trapezoid)

**Volume of Rectangular Prisms**
Volume = length x width x height
Simply multiply these 3 dimensions.

\[ V = 3 \times 10 \times 2 = 60 \text{ m}^3 \]

Remember a cube has all 3 dimensions congruent.

\[ 2 \times 2 \times 2 = 8 \text{ cm}^3 \]

**Volume of Cylinders**
Use the formula: \( V = \pi r^2 h \) where \( \pi = 3.14 \) \( r \) = radius of the circular base \( h \) = height of the cylinder

\[ V = \pi r^2 h \]

\[ V = 3.14 \times 3^2 \times 8 \]

\[ V = 3.14 \times 9 \times 8 \]

\[ V = 226.08 \text{ in}^3 \]

\[ V = \pi r^2 h \]

\[ V = 3.14 \times 4^2 \times 3 \]

\[ V = 3.14 \times 16 \times 3 \]

\[ V = 15.072 \text{ m}^3 \]

**Change in Perimeter**
This is similar to change in volume. **Draw the original shape and then draw the new shape.** Write in the numbers according to the problem and then solve.

Mary drew a picture that was 8 inches wide and 10 inches long. In order to display the picture on her wall, her mother enlarged it by 2 times the length and 2 times the width. What is the perimeter of the new picture?

\[ \text{Original perimeter} = 10 + 10 + 8 + 8 = 20 + 16 = 36 \text{ in} \]

\[ \text{New perimeter} = 20 + 20 + 16 + 16 = 40 + 32 = 72 \text{ in} \]

**Change in Area**
This is similar to change in perimeter and change in volume. It is easiest if you draw the original shape and then draw the new shape with the new dimensions. Find the area of both shapes and compare.
In the diagram below, figure 1 is a square and figure 2 is a rectangle. How does the area of figure 1 compare to the area of figure 2?

A) the area of figure 1 is twice the area of figure 2
B) the area of figure 1 is one-half the area of figure 2
C) The area of figure 1 is one-third the area of figure 2
D) the area of figure 1 is one-fourth the area of figure 2

To solve, find the area of figure 1 (A=bh, A=7·7 = 49) Find the area of figure 2 (A=bh, A=14·7=98). Compare 49 to 98 and you will see that 49 is half of 98. therefore, the answer is B.

**Change in Volume**

When you are told of the volume of a rectangular prism and then given a change in one or more of the dimensions, draw a picture of the original prism and the new one. Find the volume of both prisms and then compare.

Example: A company was trying to find a better box to hold more of their product when shipping it to the local stores. The original box had a width of 6 inches, a length of 10 inches, and a height of 5 inches. The designers of the new box decided to double all the dimensions. What is the volume of the new box? How much larger is the new box than the original box?

First, draw the original box and then the new box. Change the dimensions as the problem suggests.

Original = 5x6x10 = 30x10 = 300in$^3$
New = 10x12x20x = 120x20 = 2400in$^3$

The new box is 8 times larger than the original (2400 ÷ 300 = 8)

Example:

Original:

\[ V = \pi r^2 h \]

\[ V=3.14 \times 3^2 \times 6 \]

\[ V=3.14 \times 9 \times 6 \]

\[ V=169.56 \text{in}^3 \]

New:

\[ V = \pi r^2 h \]

\[ V=3.14 \times 1^2 \times 3 \]

\[ V=3.14 \times 1 \times 3 \]

\[ V=9.42 \text{in}^3 \]

\[ 169.56 \div 9.42 = 18 \]

The new cylinder is 18 times smaller than the original.

**Circles**

Follow the formulas for whatever you need. Remember your vocabulary:

- **radius**
- **diameter**
- **circumference**
- **area**

Hint: Don’t forget to look carefully at the units in the problem and the units they are asking for in the answer.

Example: Paul had a small wading pool in his backyard. It had a radius of 10 ft. He wanted to replace it with a larger in-ground pool. He measured his backyard and found that he had enough room for a pool with a diameter of 60 ft. What would be the circumference of his new in-ground pool? How much bigger would the area of the new pool be?

Draw the original wading pool and the new in-ground pool
Label the radius of each.
The first question asks for the circumference of the new pool. Simply write the circumference formula and substitute the radius in for \( r \):

\[
C = 2\pi r \\
C = 2 \cdot 3.14 \cdot 30 \\
C = 3788.4 \text{ ft}
\]

The second question asks how much larger the area would be on the new pool. Simply find the area of the original wading pool and the area of the new in-ground pool and compare:

Wading: \( A = \pi r^2 \)

- \( A = \pi 10^2 \)
  - \( A = 3.14 \cdot 100 \)
  - \( A = 314 \text{ ft}^2 \)

In-ground:

- \( A = \pi r^2 \)
  - \( A = \pi 30^2 \)
  - \( A = 3.14 \cdot 900 \)
  - \( A = 2826 \text{ ft}^2 \)

\[
2826 \div 314 = 9
\]

The area of the in-ground pool is 9 times greater than the wading pool.

The ratio of the area of the pools is 314:2826 or 1:9

The ratio of the radii of the pools is 10:30 or 1:3

The ratio of the diameters of the pools would be 20:60 or 1:3

MA.7.G.4.1 Practice Problems

1. Find the volume of the cylinder: If the cylinder was reduced by a scale factor of 2, what would the volume become?

2. A company introduced a new box that was twice the height of the original box. What is the volume of the new box if the original box was 12 inches high, 7 inches wide, and 7 inches long?

3. Juan had a rectangular garden in his backyard that was 10 ft. long and 6 ft. wide. His father told him that he could double the width in order to plant more vegetables. What is the perimeter of his new garden?

4. Juan had a rectangular garden in his backyard that was 10 ft. long and 6 ft. wide. His father told him that he could double both the width and length in order to plant more vegetables. What is the area of his new garden?

5. A company had a box that was 4m long, 3m wide, and 2m high. They created a new box that was double the length and width, but the height remained the same. What is the volume of the new box? How much larger is the new box than the original box?

6. You have two circles with circumference \( \pi \) and \( 4\pi \).
   a. What is the ratio of the areas of the circles?
   b. What is the ratio of the diameters?
   c. What is the ratio of the radii?

7. Jesse used 1-foot square tiles to cover the floor of his 6-foot-by-6-foot bathroom and wants to use the same tiles in the kitchen. The floor of his kitchen is double the length and double the width of the bathroom floor. How many times the number of floor tiles used to cover the bathroom floor should it take to cover the kitchen floor?

8. The cover of a calendar is printed on a sheet of paper that measures 60 cm by 30 cm. The diagonal of this sheet of paper is 67.1 cm. If a smaller version of this calendar is printed on a sheet of paper with \( \frac{1}{4} \) the area, by what factor would the length of the diagonal decrease?
   A. \( \frac{1}{4} \)  
   B. \( \frac{1}{3} \)  
   C. \( \frac{1}{2} \)  
   D. 2

9. A store that specializes in realistic miniature furniture wants to model a circular end table. The company wants to reduce the diameter of the tabletop by a factor of \( \frac{1}{2} \). How is the area of the tabletop affected?
   A. It is reduced by a scale factor of \( \frac{1}{4} \)
   B. It is enlarged by a scale factor of 4
   C. It is reduced by a scale factor of \( \frac{1}{2} \)
   D. It is enlarged by a scale factor of 2

10. Wilma made a decorative piece shaped like a square pyramid with the dimensions shown.

\[
\text{Volume of a pyramid:} \quad V = \frac{1}{3} Bh
\]

where \( B \) is the area of the base and \( h \) is the height.

She wants to double the volume of the piece. Which of the following square pyramid pieces will have a volume that is twice the volume of Wilma’s decorative piece?
11. Daniel is designing and building a small storage shed. He wants the dimensions of the shed to be one half the dimensions of the shed shown below.

If the dimensions of the shed above are each divided in half, the volume of Daniel’s new storage shed will be what fraction of the volume of the original storage shed?

12. The Gordons decided to enlarge their circular fishpond. The original pond had a diameter of 7 feet. The enlarged pond has a radius of 14 feet.

How many times the circumference of the original pond is the circumference of the enlarged pond?